Risk and Return in Production

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This paper investigates the nature of the risk-return tradeoff facing firms. The model is derived from a firm's intertemporal profit maximization condition under uncertainty with the short-run variable cost function and is framed in the context of a production-based asset pricing model. We identify output growth, the investment-capital ratio, variable input price growth, and technology shocks as the fundamental factors governing the risk-return tradeoff over time and across equities. We find that the investment-capital ratio captures the influence of business cycle fluctuations as well as the role of firm size and the book-to-market ratio in equity returns. Finally, we examine a joint link between capital investment and equity returns using Tobin's Q, and show that while investment determines equity returns, it responds negatively to changes in the risk premium.

Key Words: Risk-return Tradeoff, Equity Premium, Investment, The Stochastic
Discount Factor, Short-run Variable Cost Function

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Ⅰ. Introduction

Risk taking is a fact of life in an uncertain world. Higher risk is often associated with a higher return. Consider small businesses or entrepreneurial ventures. They are a sizeable and growing part of the economy, and account for much of the capital stock, employment, and a surprisingly large fraction of innovations in most U.S. industries [Brock and Evans (1987)]. These firms are young and growing, and tend to invest in fertile activities that generate higher rates of profits or returns than large firms. Entrepreneurial activities, however, carry with them substantial risks. Small firms are likely to be highly leveraged and may face a higher probability of default and failure than large firms [Brock and Evans (1987)]. Moreover, because small firms operate on a smaller scale, their production and hence profits are more sensitive to shocks. To the extent that this volatility cannot be diversified away, this should translate into a higher risk premium for small firms than for large firms. These considerations suggest that firms often trade risk for higher profits or returns.

This study investigates the nature of the risk-return tradeoff facing entrepreneurship and firms. While there are earlier studies analyzing this issue in asset pricing models [Chan and Chen (1991), Fama and French, (1992, 1993), Daniel and Titman (1997), Perez- Quiros and Timmermann (2000)], they lack theoretical underpinnings of firm behavior and there is little formal knowledge about the risk-return tradeoff inherent in the commitment of capital to new high risk ventures. In fact, these studies are silent about the forces that determine the risk-return tradeoff, and many variables used to explain equity returns are chosen largely on the basis of goodness-of-fit rather than the directives of a well developed theory. This research intends to redress the shortcomings of earlier studies by proposing a microfoundational framework to address many issues in the risk- return tradeoff facing firms.

The model is derived from a firm's intertemporal profit maximization condition under uncertainty, with the short-run variable cost function by taking physical capital as a quasi-fixed input. It is framed in the context of a production-based asset pricing model and belongs to a class of the stochastic discount factor model that is widely used in modern financial

research (see the Appendix for a brief summary). The model allows us to find the risk factors determining equity premium or returns. With adjustments costs in physical capital, we can also analyze a firm's capital investment decision and examine a joint link that lies between physical investment and equity returns. We identify output growth, capital stock growth or the investment-capital ratio, variable input price growth, and technology shocks as the fundamental risk factors governing the risk and return relationship over time and across equities. Of particular note in this finding is the investment-capital ratio. Cochrane (1991, 1996) shows that investment growth rates or investment-capital ratios determine equity returns [Li et al. (2004)]. The present model therefore lends support to Cochrane's use of the investment-capital ratio, but there are other relevant risk factors determining expected returns such as output growth, variable input price growth, and technology shocks. We then examine the Hansen-Jaganathan volatility bounds [Hansen and Jaganathan (1991)] in the context of the production-based asset pricing model. They are typically derived with consumption- based models [Campbell et al. (1997), Chochrane (2001)].

Identifying the risk factors that can explain the cross-sectional variation in equity returns continues to be a central issue in finance. The proposed model is particularly useful to address this issue because it is based on a firm's optimization behavior. Earlier studies find that the expected return is negatively associated with firm size [Banz (1981), Chan and Chen (1991), Fama and French (1992), Daniel and Titman (1997)]. Fama and French (1993) demonstrate that market portfolio, firm size, and the book-to-market ratio explain the cross-sectional variation in stock returns. We show that these variables constitute proxies for exposure to the underlying economic factors identified in this paper. In particular, expected returns are negatively related to firm size because small firms have more potential to exploit economies of scale and are more exposed to production risks than are large firms. Further, we find that the investment-capital ratio captures the role of both firm size and the book-to-market ratio in equity returns. The proposed model is a linear multifactor model for asset pricing where the fundamental risk factors themselves are economic, state variables that are now identified from first principles rather than out of goodness-of-fit considerations to explain the time series as well as cross sectional pattern of expected equity returns.

We solve a firm's capital investment decision problem by allowing for adjustment costs of capital and examine the joint behavior between equity returns and physical investment. In traditional investment models, future cash flows are usually discounted at a constant interest rate and the effect of time-varying discount rates is ignored. Thus physical investment responds only to risk-free constant interest rates, implying that capital investment and equity returns are independent of each other. This is a reflection of the long-standing intellectual division between macroeconomics and finance, but is not a valid assumption in an uncertain, stochastic environment. We allow the discount rate to vary stochastically over time and examine how Tobin's Q and hence investment are related to expected future returns. An important finding of this study is that a firm's intertemporal equilibrium condition helps determine the stochastic discount factor and hence equity returns, but the investment decision is driven by adjustment costs. In earlier studies [Cochrane (1991)], in contrast, both optimal investment and equity returns are driven by adjustment costs. We find that physical investment responds negatively to changes in the risk premium or expected future equity returns. However, according to the asset pricing equation, investment determines equity returns, so there is a clear simultaneous or joint link between equity returns and investment. This simultaneity is not well recognized in the investment and asset pricing literature. The paper's results carry implications for risk analysis, cost of capital calculations and investment analysis, the equity premium puzzle, and other applications.

II. The Risk-Return Tradeoff Facing Firms as Reflected in the Stock Market: A Production-based Model of Asset Pricing with the Short-run Variable Cost Function

The cost function summarizes a firm's optimizing behavior. As such, this function indirectly reveals its risk behavior as well.[1] A firm operating in

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[1] This idea is in stark contrast to the traditional analysis of risk taking firms. This analysis uses the utility function to describe a firm's risk behavior. See Moschini and Hessessy (2000) for an extensive discussion of a firm's risk behavior as applied to agriculture.

a region of declining marginal costs will react very differently to a change in demand than a firm operating in a region of rising marginal costs. Figure 1 illustrates this point with the total cost curve.[2] A concave portion of the curve occurs at low levels of output, while a convex portion occurs at high levels of output. This kind of the total cost curve produces the standard U-shaped average and marginal cost curves. The concave portion of the total cost curve corresponds to a region of declining marginal costs, and the convex portion is associated with a region of rising marginal costs. Suppose that a firm operates in the concave portion of the total cost curve or in a region of declining marginal costs. If the firm produces A units each period, its average cost per unit is C_1. If, however, the firm varies output between producing $A - \varepsilon$ units in one period and $A + \varepsilon$ units in the next period, its average cost per unit is C_2. By standard arguments using Jensen's inequality for a concave function, the latter strategy makes lower cost. Thus the firm will find it optimal to allow production to vary more than demand or sales. In contrast, a firm operating in the convex portion of the total cost curve or in a region of rising marginal costs will find it optimal to keep production constant. In Figure 1, if demand or sales varies between $B - \varepsilon$ and $B + \varepsilon$ units, the firm can reduce cost by producing B units each period instead of varying output by arguments using Jensen's inequality for a convex function. Inventories reduce fluctuations in demand by allowing a firm to produce a constant flow of output.
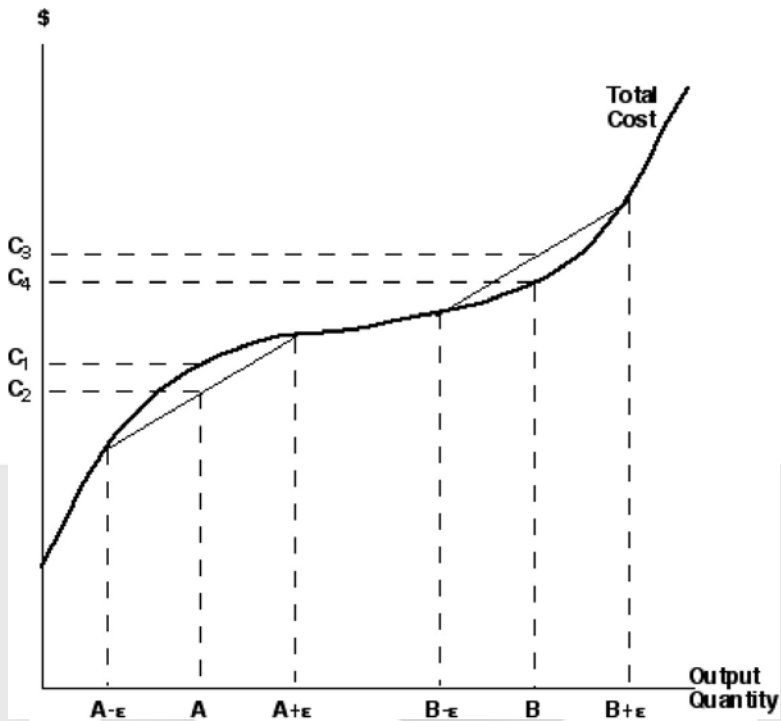
From the above result, we have

DEFINITION. *The degree of production risk* (κ) *can be measured from a firm's cost function and is defined by*

$$\kappa \equiv - \frac{\partial MC_t}{\partial y_t} \frac{y_t}{MC_t} \equiv - \frac{\partial \ln MC_t}{\partial \ln y_t} \tag{1}$$

where y_t *is the output level at time* t *and* MC_t *is the marginal cost of output at time* t.

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2) The ensuing discussion draws on Ramey (1991).

<Figure 1> Cost curve and the variability of produciton

Then we can state

PROPOSITION 1. *If the degree of production risk is negative, i.e., $\kappa < 0$, marginal costs are rising and a firm finds it optimal to keep production constant and hence to smooth production. If, on the other hand, the degree of production risk is positive, i.e., $\kappa > 0$, marginal costs are declining and a firm finds it optimal to vary production. In this case, a firm experiences volatility of production and is therefore bearing a high risk.*

We can relate the degree of production risk to the extent of returns to scale, which is measured from the slope of the average cost curve.

RESULT 1. *The degree of production risk is related to the degree of returns to scale. When there are increasing returns to scale or economies of scale, the degree of production risk is positive, while the degree of production risk is negative with decreasing returns to scale or diseconomies of scale.*

   Importantly, a firm's risk behavior impinges on the risk premium of its equity. If a firm operates in a region of declining marginal costs or increasing returns to scale, it is facing a high risk because of the possibility of production volatility and investors need to be compensated with a higher return to bear the risk. This implies that the risk premium of a firm's equity mirrors its risk behavior about demand and cost conditions. This idea provides the catalyst for the production-based CAPM (Capital Asset Pricing Model) developed by Kim (2003). Formally, in the production-based asset pricing model, the stochastic discount factor is equal to a firm's intertermporal marginal rate of transformation or substitution in output supply represented by the ratio of discounted marginal costs in two periods. The excess return that a firm must earn, and hence offer to its investors relative to the risk-free return, depends on the covariance with the marginal cost of output or the ratio of discounted marginal costs. If a firm's equity offers a return that is negatively correlated with the marginal cost, then it must command a higher return. In particular, a firm wants to have a higher output level if its price increases. On the other hand, if the marginal cost of this increase in output is large, this means that the firm is bearing a high risk and it needs to be compensated. In this case, investors will expect a higher return.

   In Kim's (2003) model, all inputs are considered variable and are assumed to adjust fully to their optimizing levels within a given period. This assumption may be reasonable for most inputs, but not for physical capital because of the presence of adjustment costs. In fact, physical capital is a quasi-fixed input-fixed in the short run but variable in the long run. Treating physical capital as a quasi-fixed input instead of a variable input is more realistic and allows us to examine many issues that are not possible with Kim's (2003) model. In particular, it allows us to derive the investment-capital ratio, which is shown to be a key risk factor explaining the cross sectional variation in equity returns (see Section 3). It also helps

us to analyze a firm's capital investment decision and to examine a joint linkage between and capital investment and equity returns (see Section 4).

We consider two sets of inputs: variable inputs (such as labor, energy, and materials) and physical capital as a quasi-fixed input. Firms act as price takers in input markets and face the non-competitive output market. Adjustment costs are separable from production costs. This is a common assumption in the literature [Hubbard (1998)]. Since adjustments costs in physical capital determine a firm's capital investment decision, this assumption allows us to separate a firm's short-run decision from its long-run investment or capital decision. Here we analyze a short-run decision facing a firm with a given capital stock. We will incorporate adjustment costs into a firm's full optimization problem and discuss its investment decision in Section 4.

Then according to the duality principle [Chambers (1988)], we can define a firm's discounted variable cost function as the solution to minimizing the short-run variable costs of producing a given level of output subject to a predetermined capital stock:

$$\widehat{VC}_t = G(y_t, \widehat{\boldsymbol{w}_t}, K_t, \tau) \tag{2}$$

where $\widehat{VC}_t$ is the discounted expenditure on variable inputs at time t (t =1, 2, $\cdots$), y_t is the level of output at time t, $\widehat{\boldsymbol{w}_t}$ is a vector of discounted variable input prices at time t whose elements are $\widehat{w}_{jt}$ ($j = 1, ..., J$), K_t is the stock of physical capital at the beginning of time t, and τ_t denotes the technology level or shock at time t.

The short-run variable cost function (2) is well known in static analysis except that input prices are discounted [Brown and Christensen (1981), Berndt and Fuss (1986)]. The cost function is discounted because a firm makes a production decision in the current period considering all future events. It has the following regularity properties: it is linear homogenous in $\widehat{\boldsymbol{w}_t}$, increasing in $\widehat{\boldsymbol{w}_t}$ and y, decreasing with respect to K; it is also concave in $\widehat{\boldsymbol{w}_t}$ and convex in K. Moreover, application of Shephard's

lemma gives the variable input demands:

$$\frac{\partial G(y_t, \widehat{\boldsymbol{w}_t}, K_t, \tau_t)}{\partial w_{jt}} = x_j(y_t, \widehat{\boldsymbol{w}_t}, K_t, \tau_t) \quad (j = 1, ..., J) \qquad (3)$$

where x_j is the demand for the jth $(j = 1, ..., J)$ variable input. $\partial G(y_t, \widehat{\boldsymbol{w}_t}, K_t, \tau_t) / \partial K_t < 0$, which measures the reduction in variable costs due to an increase in capital stock.

To derive implications for equity returns, we consider the familiar Cobb-Douglas form for the variable cost function (2):

$$G(y_t, \widehat{\boldsymbol{w}_t}, K_t, \tau_t) = y_t^{\phi_y} \Pi_{j=1}^{J} (\widehat{w}_{jt} / \tau_t)^{\phi_j} K_t^{\phi_K} \qquad (4)$$

where ϕ's are parameters to be estimated.[3] According to Shephard's lemma (3) applied to (4), $\partial \ln G(\cdot) / \partial \ln \widehat{w}_{jt} = \phi_j > 0$, which is the share of the jth $(j = 1, ..., J)$ input in total variable cost. The linear homogeneity of the cost function implies that $\sum_{j=1}^{J} \phi_j = 1$. Technical change is assumed to be Hicks neutral. We also have

$$\frac{\partial G(y_t, \widehat{\boldsymbol{w}_t}, K_t, \tau_t)}{\partial K_t} = \phi_K y_t^{\phi_y} \Pi_{i=1}^{n} (\widehat{w}_{jt} / \tau_t)^{\phi_j} K_t^{\phi_K - 1} \qquad (5)$$

The condition $\partial G(\cdot) / \partial K_t < 0$, together with the convexity of (4) with respect to K, implies that $\phi_K < 0$. It turns out that $\partial \ln G(\cdot) / \partial \ln K_t = -\phi_K > 0$, which is the share of capital in total variable cost. The parameter ϕ_y is related to the degree of short-run returns to scale. The degree of short-run returns to scale derived with the capital

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3) The specification of a technology shock in the cost function is not the same as that in the production. A technology shock affects the production function directly, but it affects the cost function negatively. Thus it enters into the cost function in an inverse form.

stock held constant is measured from the slope of the average variable cost curve and is defined by $RTS_t = 1 - \partial \ln G(y_t, \widehat{\boldsymbol{w_t}}, K_t, \tau_t) / \partial \ln y_t$ [Christensen and Greene (1976)]. When $RTS > 0$, short-run increasing returns to scale or economies of scale exist; $RTS < 0$ implies short-run decreasing returns to scale or diseconomies of scale, and $RTS = 0$ implies short-run constant returns to scale. For (4), the degree of short-run returns to scale is equal to $1 - \phi_y$.

From (4), we obtain short-run marginal cost as

$$\frac{\partial G(y_t, \widehat{\boldsymbol{w_t}}, K_t, \tau_t)}{\partial y_t} = \phi_y \, y_t^{\phi_y - 1} \, \Pi_{i=1}^{n} (\widehat{w}_{jt} / \tau_t)^{\phi_j} K_t^{\phi_K} \tag{6}$$

From (6), we find that $-\partial \ln(\partial \ln G(\,\cdot\,)/\partial y_t)\,\partial \ln y_t = 1 - \phi_y$. Thus the degree of production risk is equal to the degree of returns to scale. We derive the stochastic discount factor as the ratio of two successive marginal costs:

$$M_{t+1} \equiv \frac{\partial G(y_{t+1}, \widehat{\boldsymbol{w_{t+1}}}, K_{t+1}, \tau_{t+1}) / \partial y_{t+1}}{\partial G(y_t, \widehat{\boldsymbol{w_t}}, K_t, \tau_t) / \partial y_t}$$
$$= \left(\frac{y_{t+1}}{y_t}\right)^{\phi_y - 1} \Pi_{j=1}^{J} \left(\frac{\widehat{w}_{jt+1} / \tau_{t+1}}{\widehat{w}_{jt} / \tau_t}\right)^{\phi_j} \left(\frac{K_{t+1}}{K_t}\right)^{\phi_k} \tag{7}$$

which is a nonlinear function of output growth, variable input price growth (adjusted by a technology shock), and capital stock growth. For analytical tractability, we take a first- order Taylor approximation of this function to obtain the following linear discount factor equation:

$$M_{t+1} \approx 1 + (\phi_y - 1)\Delta \ln y_{t+1} + \Sigma_{j=1}^{J} \phi_j \, \Delta \ln(\widehat{w}_{jt+1} / \tau_{t+1})$$
$$+ \phi_K \, \Delta \ln K_{t+1} \tag{8}$$

where Δ is the first difference operator. An important variable that we will consider later is $\Delta \ln K_{t+1} = (K_{t+1} - K_t)/K_t$, which is the growth rate of the capital stock or the (net) investment-capital ratio. We can now apply the stochastic discount factor model (see the Appendix) to (8) to identify the risk factors determining the equity premium or returns, which is a key concern in asset pricing. Provided that output growth, input price growth, and capital stock growth are orthogonal to one another, we have

$$
\begin{aligned}
Cov_t(r_{t+1}, M_{t+1}) = {} & (\phi_y - 1) Cov_t(r_{t+1}, \Delta \ln y_{t+1}) \\
& + \Sigma_{j=1}^{\phi_j} Cov_t(r_{t+1}, \Delta \ln(\widehat{w}_{jt+1}/\tau_{t+1})) \\
& + \phi_K Cov_t(r_{t+1,} \Delta \ln K_{t+1})
\end{aligned}
\tag{9}
$$

where r_{t+1} is the real rate of return on an asset from time t to time $t+1$ and $Cov_t(\cdot)$ denotes covariance conditional on information at time t. We then obtain:

PROSITION 2. *With the variable cost function of the Cobb-Douglas form (4), the equity risk premium is expressed as* [4)]

$$
\begin{aligned}
\frac{E_t[r_{t+1}] - r_{t+1}^f}{1 + r_{t+1}^f} = {} & - Cov_t(r_{t+1}, M_{t+1}) = (1 - \phi_y) Cov_t(r_{t+1,} \Delta \ln y_{t+1}) \\
& - \Sigma_{j=1}^J \phi_j Cov_t(r_{t+1}, \Delta \ln(\widehat{w}_{jt+1}/\tau_{t+1})) \\
& - \phi_K Cov_t(r_{t+1}, \Delta \ln K_{t+1})
\end{aligned}
\tag{10}
$$

where r_{t+1}^f is the risk-free rate of return from time t to time $t+1$, and $E_t[\cdot]$ is an expectation conditional on information available to investors at time t.

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4) A similar expression can be derived by taking a lognormal approximation. In this case, there is a variance term on the left hand side of (25) to adjust for Jensen's inequality (see Kim, 2003).

Equation (10) shows that the risk premium of a firm's equity is the sum of three conditional covariance terms. The first term is the covariance of equity returns with output growth scaled by the degree of returns to scale. The second term is the weighted sum of the covariance of equity returns with input price growth adjusted by a technology shock; the weight is given by the cost share of an input. The third term is the covariance of equity returns with capital stock growth. Note that the covariances are conditional on information available at time t and vary over time, producing a time-varying risk premium. We can expect the covariance between equity returns and output growth to be positive because a firm with growing demand will invest in anticipation of a higher return. Then, ceteris paribus, the degree of short-run returns to scale can tell us about the presence or absence of the risk premium.[5] A firm with increasing returns to scale or economies of scale $(1-\phi_y > 0)$ exhibits a positive risk premium, while a firm with decreasing returns to scale or diseconomies of scale $(1-\phi_y < 0)$ has a negative premium. In the absence of input price and capital stock growth, a firm with constant returns to scale $(1-\phi_y = 0)$ has no return differential with a risk-free asset.

In addition, since the cost share of an input is positive $(\phi_j > 0, j = 1, ..., J)$, the effect of input price growth on the risk premium depends on the sign of the covariance of equity returns with input price growth. If the covariance is positive, higher input price growth makes a risky firm more desirable, producing a negative risk premium. If the covariance is negative, on the other hand, higher input price growth makes a risky firm less desirable, producing a positive risk premium. With negative ϕ_K, the effect of capital stock growth on the risk premium depends on the covariance of equity returns with capital stock growth. If the covariance is negative, we expect that higher capital stock growth or higher investment-capital ratio will lead to a lower risk premium. These results suggest that, although there are constant returns to scale or there is production smoothing by firms (for these firms, the covariance between equity returns and output growth is zero), a (positive) risk premium can

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5) While we present the result based on short-run returns to scale, it can be applied to long-run returns to scale as well.

arise with input price and/or capital stock growth. In sum, equation (10) suggests that to the extent that different risks faced by a firm cannot be diversified away, they will translate into a higher risk premium and hence a higher equity return of a firm.

The Hansen-Jagannathan volatility bounds [Hansen and Jagannathan (1991)] are typically derived in the context of the consumption-based CAPM. We can derive the Hansen-Jagannathan volatility bounds for the production-based model. The mean of the stochastic discount factor M_{t+1} can be considered 1, i.e., $E_t[M_{t+1}] \approx 1$. Since output growth, input price growth, and capital stock growth are orthogonal to one another, the variance of the stochastic discount factor in (7) is

$$Var_t(M_{t+1}) = (\phi_y - 1)^2 \, Var_t(\Delta \ln y_{t+1}) \tag{11}$$
$$+ \sum_{j=1}^{J} \phi_j^2 \, Var_t(\Delta \ln(\widehat{w}_{jt+1}/\tau_{t+1})) + \phi_K^2 \, Var_t(\Delta \ln K_{t+1})$$

where $Var_t(\cdot)$ is variance conditional on information at time t. We can then state the following:

RESULT 2. *The Hansen-Jagannathan volatility bounds for the production-based model can be approximated by*

$$\frac{E_t[r_{t+1}] - r_{t+1}^f}{\sigma_t(r_{t+1})} \leq \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]}$$
$$\approx \sqrt{\begin{array}{l} (\phi_y - 1)^2 \, Var_t(\Delta y_{t+1}) + \sum_{j=1}^{J} \phi_j^2 \, Var_t(\Delta \ln(\widehat{w}_{jt+1}/\tau_{t+1})) \\ + \phi_K^2 \, Var_t(\Delta \mathrm{Ln}\, K_{t+1}) \end{array}} \tag{12}$$

where $\sigma_1(M_{t+1})$ and $\sigma_1(r_{t+1})$ are standard deviations of M_{t+1} and r_{t+1}, and $(E_t[r_{t+1}] - r_{t+1}^f)/\sigma_t(r_{t+1}))$ is the Sharpe ratio. In particular, when variable input prices are constant and the capital stock grows constant over time, the Hansen-Jagannathan volatility bounds are given by

$$\frac{E_t[r_{t+1}] - r_{t+1}^f}{\sigma_t(r_{t+1})} \leq \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]} \approx |1 - \phi_y| \sigma(\Delta \ln y_{t+1}) \qquad (13)$$

When $\phi_y = 0$, the maximum Sharpe ratio is equal to the variability of output growth. When $\phi_y = 1$ (short-run constant returns to scale), the maximum Sharpe ratio is equal to zero.

Hansen and Jagannathan's bounds test is a very useful tool because it can be conducted as a diagnostic tool to check whether a firm's production process is consistent with some important moments of equity returns and hence to check the validity of the proposed production-based asset pricing model.

While equation (10) provides an informative framework to analyze equity returns, it can be equivalently expressed as a multifactor beta asset pricing model.

PROPOSITION 3. *Given the linear stochastic discount factor* (7), *we have the expected return-beta representation of the form:*

$$E_t[r_{t+1}] = \lambda_{0t+1} + \beta_{yt+1}\lambda_{yt+1} + \textstyle\sum_{j=1}^J \beta_{jt+1}\lambda_{jt+1} + \beta_{Kt+1}\lambda_{Kt+1} \quad (14)$$

where

$\lambda_{0t+1} \equiv r_{t+1}^f$

$\beta_{yt+1} \equiv Cov_t(r_{t+1}, \Delta \ln y_{t+1}) / Var_t(\Delta \ln y_{t+1})$

$\lambda_{yt+1} \equiv (1 - \phi_y)(1 + r_{t+1}^f) Var_t(\Delta \ln y_{t+1})$

$\beta_{jt+1} \equiv Cov_t(r_{t+1}, \Delta \ln(\widehat{w}_{jt+1}/\tau_{t+1})) / Var_t(\Delta \ln(\widehat{w}_{jt+1}))$
$\qquad (j = 1, ..., J)$

$\lambda_{jt+1} \equiv -\phi_j(1 + r_{t+1}^f) Var_t(\Delta \ln(\widehat{w}_{jt+1}/\tau_{t+1})) \ (j = 1, ..., J)$

$\beta_{Kt+1} \equiv Cov_t(r_{t+1}, \Delta \ln K_{t+1}) / Var_t(\Delta \ln K_{t+1})$

$\lambda_{Kt+1} \equiv -\phi_K(1 + r_{t+1}^f) Var_t(\Delta \ln K_{t+1})$

Here the betas (β's) are the factor loadings defined by the coefficients in a regression of equity returns on output growth $(\Delta \ln y_{t+1})$, input price growth adjusted by a technology shock $(\Delta \ln (\widehat{w}_{jt+1} / \tau_{t+1}))$, and capital stock growth or the investment-capital ratio $(\Delta \ln K_{t+1})$:

$$r_{t+1} = \beta_0 + \beta_{yt+1} \Delta \ln y_{t+1} + \sum_{j=1}^{J} \beta_{jt+1} \Delta \ln (\widehat{w}_{jt+1} / \tau_{t+1})$$
$$+ \beta_{Kt+1} \Delta \ln K_{t+1} + \varepsilon_{t+1} \tag{15}$$

where ε_{t+1} is a iid white noise. (More precisely, because output growth, input price growth, and capital stock growth are assumed to be orthogonal to one another, the betas are derived from a series of a simple regression of equity returns on each variable. Equation (15) allows for a general case that does not assume the orthogonality of the variables.) λ's are the prices of risk factors associated with output growth, input price growth, and capital stock growth. Conversely, given β's and λ's of the form in (14), we can find ϕ's such that (7) holds.

Equation (14) has the standard structure of a multifactor beta pricing model [Cochrane (2001)]. Equation (15) shows that the proposed model is a linear multifactor model with four fundamental risk factors that price equities, namely output growth, variable input price growth, capital stock growth or the investment-capital ratio, and technology shocks (their effect is subsumed in the input price growth). These variables govern the risk-return trade facing entrepreneurship or firms in the proposed production-based model. It is important to note that the betas and the risk prices are conditional on information at time t and vary over time, producing time-varying expected equity returns. Equations (10) and (14) essentially provide the same information about equity returns. In fact, one model can be derived from the other, and vice versa. However, we can gain more insight from (14) than from (10) to explain the variation in expected equity returns across firms. In particular, the predictable variation in equity returns can be driven by changes in betas and changes in risk prices. Output growth

has a positive beta $(\beta_{yt+1} > 0)$, given that the covariance between equity returns and output growth is positive. With substantial evidence on economies of scale for many firms, i.e., $(1 - \phi_y) > 0$, we expect that output growth also has a positive risk price $(\lambda_{yt+1} > 0)$. Input price growth, however, has a negative beta $(\beta_{jt+1} < 0, j = 1, ..., J)$ to the extent that an increase in input price raises the cost of production without raising the price of output; it also has a negative risk price $(\lambda_{jt+1} < 0, j = 1, ..., J)$ since $\phi_j > 0$ $(j = 1, ..., J)$. A negative beta together with a negative risk price for input price growth produces a positive risk premium for this factor. Capital stock growth or the investment-capital ratio has a positive beta $(\beta_{Kt+1} > 0)$ assuming a positive covariance between equity return and capital stock growth, and a positive risk price $(\lambda_{Kt+1} > 0)$ since $\phi_K < 0$. The expected return of an equity is high if that equity has a high beta or a large risk price of a factor. In particular, a high (low) value of returns to scale together with a high (low) output growth variability yields a high (low) risk price of output growth. Given a positive value of the beta for output growth, this in turn produces a high (low) expected equity return. From a conditional point of view, evidence shows that the time-varying risk price (λ) explains expected returns better than time-varying undiversifiable risk as measured by beta (β) [Lettau and Ludvigson (2001) for evidence from a consumption-based model].

While some of the variables identified here are considered in earlier studies, they are largely chosen atheoretically to fit the data rather than derived from the directives of a well developed theory. Of particular note is the investment-capital ratio. In Kim's (2003) model with capital taken as a variable input, the return on capital, which is approximated by the growth rate of the rental price of capital, takes the place of capital stock growth or the investment-capital ratio. In Cochrane's (1991) investment-based asset pricing model, the factors are investment returns or investment growth rates [Li et al. (2006)]. The identification of variable input price growth as a determinant of equity returns is also important. In particular, our model can explain higher oil prices associated with the Iraq war as a risk factor, leading investors to demand higher risk premium. Unfortunately, none of the earlier studies have heretofore ascertained all of the variables identified

here - output growth, the investment-capital ratio, variable input price growth, and technology shocks - as fundamental to explaining equity returns.

There is considerable evidence that excess stock returns vary over business cycles: the risk premium should be higher at the bottom of a business cycle when investors require a higher excess return to hold risky assets (stocks) [Siegel (2002)]. Also, to allow for time-varying risk premia or equity returns, business cycle variables such as real GDP or investment growth are often used as conditioning variables [Lettau and Ludvigson (2002), Cochrane (1996)]. Our model has business cycle variables built into it and therefore can explain cyclical variation in asset returns. In particular, volatility in aggregate investment spending characterizes business cycle fluctuations, and technology or productivity shocks are identified as a driving force for business cycles [Kydland and Prescott (1982)]. Because the investment-capital ratio and technology shocks determine equity returns in our model, this clearly suggests cyclical variation in equity returns.

Ⅲ. Cross-Sectional Variation in Expected Returns: What Do Fama and French's (1992, 1993) Three Factors Represent?

The paper's proposed model is couched in the context of a representative firm framework. When markets are complete, firms can be aggregated into a single representative agent, which allows us to analyze aggregate time series equity returns of an industry or the economy. The production-based asset pricing model, one the other hand, views each firm that employs a specific production technology as if it were an asset to investors that generate different rates of returns. We can allow for heterogeneous firms with different marginal costs of production by introducing different production shocks or different returns to scale. The model is, therefore, particularly, useful to analyze the cross-sectional variation in equity returns.

<Table 1> Long-Term Returns of NYSE/AMEX/Nasdaq Stocks Ranked by Size, 1926-2000*

Size Decile	Geometric Mean	Standard Deviation	Market Beta	Largest firm in Decile, Sept. 2000	Total Capitalization of Decile, Sept. 2000
Largest	10.26%	19.0%	0.91	$524.35B	$11,757B
2	11.32%	22.7%	1.04	$10.34B	$1,797B
3	10.59%	24.5%	1.09	$4.14B	$865B
4	11.52%	27.6%	1.13	$2.18B	$547B
5	11.32%	30.1%	1.16	$1.33B	$400B
6	11.31%	30.2%	1.18	$840.0M	$287B
7	10.99%	32.5%	1.24	$537.7M	$222B
8	11.27%	34.7%	1.28	$333.4M	$138B
9	12.59%	38.8%	1.34	$192.6M	$117B
Smallest	16.71%	49.3%	1.42	$84.5M	$74B

* Taken from Siegel (2002, p. 133).
  Note: B = billion, M = million.

Small stocks on average outperform large stocks, and the relationship between firm size and expected stock returns is well documented in the literature. Table 1 presents long-term compound annual returns on stocks listed on the New York Stock Exchange, the American Stock Exchange, and the Nasdaq, sorted into deciles according to their market capitalization, 1926-2000 [Siegel (2002)]. The top two deciles, or the top 20 percent of all firms, are often called *large-cap stocks* and comprise most of the Standard & Poor (S&P) 500 Stock Index. Deciles 3 through 5 are called *mid caps;* deciles 6 through 8 are called *low (or small)* caps; and the smallest 20 percent are called *micro caps*. The annual return on the smallest decile of stocks is about 6 1/2 percentage points higher than the largest decile, and the standard deviation of these small stocks is also higher. In general, there is clear evidence that the expected return is negatively associated with firm size; that is, small firms have higher expected returns than large firms do. However, the portfolio beta is not high enough for the smaller capitalization stocks to justify their extra return. This captures one

of the failures of the traditional CAPM (Capital Asset Pricing Model), known as the celebrated "small-firm effect" (Banz, 1981). There have been many attempts to account for the small firm effect, but results are not satisfactory [Chan and Chen (1991), Daniel and Titman (1997)].

Fama and French (1992, 1993) note that a firm's average stock return is related to size (market capitalization) as well as value (book value to market value ratio) [Hodrick and Zhang (2001), Siegel (2002) for recent evidence]. They find that these two variables capture much of the cross-sectional variation in stock returns. Firm with small market value have, on average, higher returns. On the other hand, firms with high book values relative to market equity have, on average, higher returns. To capture two features of average returns (the size and value effects), Fama and French (1993) posit a three- factor model in which the priced risk factors are market, size, and value factors. The market risk premium is the excess of the value-weighted market portfolio over the risk- free return (the return on a Treasury bill rate). The size factor is the difference in the return on a portfolio of small capitalization stocks and the return on a portfolio of large capitalization stocks. The value factor is the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low book-to-market stocks. Fama and French (1993) show that the three-factor-model performs well in explaining the cross-sectional variation in stock returns. They argue that the three factors may proxy for firm risk sensitivities, thus compensating investors with higher expected returns. The Fama and French model, or some extended variation of it, has become the workhorse that now dominates empirical research in asset pricing [Cochrane (2001) for a survey].

There are, however, outstanding issues with the Fama and French model. The model is empirically motivated, and it is not yet clear how their factors are related to the underlying economic risks so proxied [Li et al. (2006)]. As a result, there is a continuing debate about Fama and French's three factors. The issue is not about whether the size and value factors can explain expected returns; rather it centers on whether these factors can possibly represent or capture economically relevant nondiversifiable, and therefore aggregate, risk. Daniel and Titman (1997) suggest that it is firm

characteristics themselves, rather than the size and value factors, that seem to be related to expected returns, having little resemblance to risk. Our production-based model is theoretically based with strong microeconomic underpinnings of firm behavior, and can offer new insight to show how Fama and French's factors are related to underlying economic risks so proxied.

Small and large firms have varying size as well as varying production and risk characteristics [Cooley and Quadrini (2001)]. Evans (1987) and Hall (1987) demonstrate that the growth rate and the variability of growth of U.S. manufacturing firms are negatively associated with firm size and age. On the other hand, small firms have more potential to exploit scale economies than do large firms [Nguyen and Resnek (1991)], implying that the degree of returns to scale is negatively related to firm size. Moreover, small firms are more exposed to production risks and changes in the risk premium than large firms. In particular, small firms face greater variability in output and investment growth than large firms. We then can expect a higher risk price or premium associated with output growth and the investment-capital ratio for small firms than for large firms. Also, small firms are more susceptible to input price shocks than are large firms, again producing a higher risk price or premium associated with input price growth for small firms than for large firms. Provided that the betas are the same for small and large firms, we can then expect a greater expected return for small firms than for large firms.

In addition, we expect that firms with different sizes and book-to-market ratios have different investment behavior. The book-to-market ratio is the inverse of Tobin's (average) q, which is equal to the ratio of the market value of a firm to the replacement cost of capital. As will be shown in the next section, because the investment-capital ratio is positively related to Tobin's q, this means that the book-to-market ratio is inversely related to the investment-capital ratio. The investment-capital ratio, in effect, contains the same information as the book-to-market ratio. Xing (2008) provides evidence for U.S. manufacturing and non-manufacturing firms that firm size as well as the book-to- market ratio are related to the investment-capital ratio. Size is negatively related to the investment-capital ratio after

controlling for the book-to-market ratio. That is, small firms have higher investment-capital ratios than large firms with similar book-to-market ratios. The book-to-market ratio is also negatively related to the investment-capital ratio after controlling for firm size. That is, firms with low book-to-market ratios have higher investment-capital ratios than firms of similar size with higher book-to-market ratios. To encapsulate these results, we have

PROPOSITION 4. *Firm size and the book-to-market ratio are negatively associated with the investment-capital ratio. This suggests that the investment-capital ratio captures the role of both firm size and the book-to-market ratio, the two variables that are shown to capture much of the cross-sectional variation in stock returns.*

Cochrane (1996) uses two components of aggregate investment - residential and nonresidential investment - and demonstrates that the investment-based model performs considerably better than the consumption-based CAPM and about as well as the traditional CAPM. Li, Vassalou, and Xing (2006) use aggregate sector investment growth rates, including household sector investment that is largely residential, as risk factors and find that their model can price the size and value portfolios at least as well as Fama and French's three-factor model to account for a large fraction of the cross-sectional variation in equity returns. While previous findings provide strong evidence for the investment-based asset pricing model, our analysis shows that there are other state variables in explaining the cross sectional variation in equity returns. In particular, since each firm is assumed to face the same input prices, there is no variation in input price growth across firms. However, firms face different technology shocks with different output levels. Then from our model, we can argue that output growth, the investment- capital ratio, and technology shocks are the fundamental risk factors determining the cross sectional variation in equity returns.

Notably, our model does not include portfolio returns considered in the traditional CAPM. Cochrane (2001) argues that asset pricing models that use portfolio returns leave unanswered the question of what explains the return-based factors. Further, according to Merton (1973), variables that

predict market returns should show up as risk factors that explain the cross-sectional variation in average returns. We have identified these variables from the directives of a firm's intertemporal equilibrium condition to explain the time series as well as cross sectional pattern of expected stock returns. The variables identified here are considered state variables or sources of priced risk in the spirit of a multifactor or intertemporal capital asset pricing model of Merton (1973). We can argue that these risk factors are orthogonal to the overall market return. Then Fama and French's three factors are indicator variables for exposure to the underlying economic factors identified in this study - output growth, the investment-capital ratio, and technology shocks.

Ⅳ. Adjustment Costs, Tobin's Q, and the Joint Link between Capital Investment and Equity Returns

In the asset pricing model (see (10) or (14)), investment - more precisely, the investment-capital ratio - (and the output level) determines equity returns, but it is treated as exogenous. Investment, however, is an endogenous choice variable in a firm's optimization problem. Analysis of a firm's investment requires the specification of an adjustment cost function $H(I_t, K_t, \omega_t)$ that is a function of gross investment I_t, the stock of physical capital K_t, and an exogenous shock ω_t. The adjustment cost function is linear homogeneous in I_t and K_t, increasing in I_t but decreasing in K_t, and convex in I_t and K_t.

A firm's problem is to maximize the market value of equity, which is equal to the expected present value of all future profits or cash flows to its shareholders, by choosing the optimal levels of output, variable inputs, and investment, subject to the capital accumulation constraint with adjustment costs. Firms face perfectly competitive markets with respect to variable inputs but a non-competitive output market. The optimization problem can be solved in a two-step procedure.[6] In the first stage, the firm

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6) See Chirinko (1993) and Hubbard (1998) who use the production function framework rather than the cost function used in this study. All of these studies are, however, based on a constant interest rate and do not consider the link between equity returns and investment.

determines the optimal choice of output and investment across periods by maximizing the expected present value of cash flows in the presence of the costs of adjustment, conditional on the given quantities of variable inputs. In the second stage, the short-run variable costs are minimized to choose the optimal input quantities at each period subject to a production constraint with the chosen output level. The second-stage problem is summarized by the short-run variable cost function (19). Given this cost function, the first-stage problem is:

$$\max_{\{y_s,\, I_s\}_{s=t}^{\infty}} E_t\left[\sum_{s=t}^{\infty} \Pi_{v=s}^{t} (1+\rho_{v+1})^{-s}\right.$$
$$\left.\left\{p_s(y_s)y_s - G(y_s,\, \boldsymbol{w_s},\, K_s,\, \tau_s) - p_s^K I_s - H(I_s,\, K_s,\, \omega_s)\right\}\right] \tag{16}$$

subject to

$$K_{s+1} = (1-\delta)K_s + I_s \ (s=t,\, t+1,\, \ldots) \tag{17}$$

with a given initial capital stock $K_t = K_t^0$. Here p is the price of output at time $s\,(s=t,\,t+1,\,\ldots)$, $\boldsymbol{w_s}$ is a vector of undiscounted nominal variable input prices at time s, p_s^K is the price of capital goods at time s, δ is the constant rate of depreciation of the capital stock, and ρ_{v+1} is the nominal rate of return on equity required by investors that is used as a discount rate between time v and $v+1$. Uncertainties arise because both future output and input prices as well as technology and adjustment cost shocks make a firm's profitability unknown, and future rates of return are unknown at the time of decision making. Note that, in the above formulation of the intertemporal profit maximization problem, the undiscounted cost function $G(y_s,\, \boldsymbol{w_s},\, K_s,\, \tau_s)$, rather than the discounted cost function $G(y_s,\, \widehat{\boldsymbol{w_s}},\, K_s,\, \tau_s)$, is used in accordance with the convention in the literature [Chirinko (1993), Pindyck and Rotemberg (1983)]. The linear homogeneity of the cost function allows us to convert the undiscounted cost function into the discounted cost function such that $G(y_s,\, \widehat{\boldsymbol{w_s}},\, K_s,\, \tau_s) = G(y_s,\, \boldsymbol{w_s},\, K_s,\, \tau_s)/(1+\rho_{s+1})$.

The first-order conditions for output and investment in the above intertemporal profit maximization problem are

$$E_t\left[\hat{p}_s\left(1+\frac{1}{\eta}\right)\right] = E_t\left[\frac{\partial G(y_s, \widehat{\boldsymbol{w}_s}, K_s, \tau_s)}{\partial y_s}\right] \quad (s = t, t+1, ...) \quad (18)$$

and

$$p_t^K + \frac{\partial H(I_t, K_t, \omega_t)}{\partial I_t} = \mu_t^K \tag{19}$$

where $\hat{p}_t$ is the discounted price of output at time t, η is the price elasticity of the demand for output that is constant over time, and μ_t^K is the imputed value or shadow price of capital to a firm of an additional unit of installed capital at time t. Equation (18) is the profit maximization condition across time. In particular, when $s = t$ (current period), this equation is the familiar static profit maximization condition at which the price of output equals marginal cost, i.e., $p_t = \partial G(y_t, \boldsymbol{w}_t, K_t, \tau_t)/\partial y_t$, for a competitive firm. The right hand expression of (18) exploits the linear homogeneity of the cost function to convert the undiscounted cost function $G(y_s, \boldsymbol{w}_s, K_s, \tau_s)$ into the discounted cost function $G(y_s, \widehat{\boldsymbol{w}_s}, K_s, \tau_s)$. Equation (19) is the first-order condition for optimal investment and shows that adjustment costs drive a firm's investment decision. Moreover, the first-order condition for capital describes the evolution of μ_t^K, the shadow value of capital, according to

$$\mu_t^K = E_t\left[\frac{-\partial G(y_{t+1}, \boldsymbol{w}_{t+1}, K_{t+1}, \tau_{t+1})/\partial K_{t+1} - \partial H(I_{t+1}, K_{t+1}, \omega_{t+1})/\partial K_{t+1} + (1-\delta)\mu_{t+1}^K}{1+\rho_{t+1}}\right] \tag{20}$$

With the transversality condition:

$$\lim_{T\to\infty} E_t\left[\Pi_{v=t}^{t+T}\left(\frac{1-\delta}{1+\rho_{v+1}}\right)\mu_{t+T+1}^K K_{t+T+1}\right] = 0 \tag{21}$$

the solution of (20) yields

$$\mu_t^K = E_t \left[\sum_{s=t}^{\infty} \Pi_{v=t}^s \left(\frac{1-\delta}{1+\rho_{v+1}} \right) \left(-\frac{\partial G(y_s, \boldsymbol{w_s}, K_s, \tau_s)}{\partial K_s} - \frac{\partial H(I_s, K_s, \omega_s)}{\partial K_s} \right) \right] \quad (22)$$

This equation states that the shadow price of installed capital is equal to its expected present discounted value of the stream of the marginal benefits of a unit of capital installed at time s, which consists of marginal cost savings generated from the reduction in both variable costs $(-\partial G(y_s, \boldsymbol{w_s}, K_s, \tau_s)/\partial K_s > 0)$ and adjustment costs $(-\partial H(I_s, K_s, \omega_s)/\partial K_s > 0)$ made possible by an additional unit of installed capital.

The profit-maximization condition (18) holds for current as well as future periods. It then implies that, between two periods t and $t+1$, the following intertemporal equilibrium condition must hold:

$$\frac{\partial G(y_t, \widehat{\boldsymbol{w_t}}, K_t, \tau_t)}{\partial y_t} = E_t \left[(1+r_{t+1}) \frac{\partial G(y_{t+1}, \widehat{\boldsymbol{w_{t+1}}}, K_{t+1}, \tau_{t+1})}{\partial y_{t+1}} \right] \quad (23)$$

or

$$E_t \left[(1+r_{t+1}) \frac{\partial G(y_{t+1}, \widehat{\boldsymbol{w_{t+1}}}, K_{t+1}, \tau_{t+1})/\partial y_{t+1}}{\partial G(y_t, \widehat{\boldsymbol{w_t}}, K_t, \tau_t)/\partial y_t} \right] = 1 \quad (24)$$

where $1+r_{t+1} = (1+\rho_{t+1})p_t/p_{t+1}$ such that $r_{t+1} \approx \rho_{t+1} - \pi_{t+1}$ with $\pi_{t+1} = \ln(p_{t+1}/p_t)$ being the rate of increase in the price of output. Equation (23) is a generalization of the intertemporal equilibrium condition in production under certainty which requires that the risk-free real interest rate is equal to the intertemporal marginal rate of transformation or substitution given by the ratio of two marginal costs, and is derived under the condition that markets are complete. Existing studies on investment or dynamic input demands employ the undiscounted, rather than the discounted, cost function and show that the static profit maximization rule holds across time. As a result, none of them accounts for the intertemporal equilibrium condition in analysis or estimation [Chirinko (1993), Pindyck

and Rotemberg (1983)]. This is not a valid analysis because a firm makes a decision in the current period considering all future events; hence the output price and the cost or input prices have to be discounted.

Importantly, equation (23) is a stochastic discount factor equation (see the Appendix) where the stochastic discount factor is defined by the ratio of two marginal costs (see equation (7)). Thus the intertemporal equilibrium condition (23) helps us to determine the stochastic discount factor and therefore equity returns. This implies that equity returns and profits or production are interrelated. In traditional analyses of investment, however, the risk-free real interest rate serves as a discount rate that is assumed to be constant and distributed independent of a firm's profits, implying that production and equity returns are independent of each other. This assumption reflects the long-standing intellectual division between macroeconomics and finance, but is not valid in an uncertain, stochastic environment. The discount rate is not fixed but varies over time, and affects the stream of a firm's profits and hence production through optimal investment. In particular, since $\rho_{t+1} = r_{t+1} + \pi_{t+1}$ (which is known as the Fisher equation), a firm's nominal equity returns are determined by real returns and the rate of increase in the price of output. The rate of increase in the price of output is determined by the profit maximization condition where marginal revenue is equal to marginal cost. Further, production conditions clearly determine real equity returns as expounded in Section 2. Thus the discount rate or equity returns vary stochastically over time and are not independent of production.

Equation (19), together with (20), determines a firm's investment decision once the adjustment cost function is specified. A convenient form that has desirable properties of the adjustment cost function is a quadratic function [Hubbard (1998)]:

$$H(I_t,\, K_t,\, \omega_t) = \frac{\theta}{2}\,(\,\frac{I_t}{K_t} - \gamma - \omega_t)^2\, K_t \qquad (25)$$

where θ and γ are parameters to be estimated. Given this adjustment cost function, the solution of (19) yields an investment specification:

$$\frac{I_t}{K_t} = \gamma + \frac{1}{\theta}\left(\mu_t^K - p_t^K\right) + \omega_t \tag{26}$$

The investment-capital ratio (I_t / K_t) is equal to the growth rate of the capital stock, i.e., $\Delta \ln K_{t+1} = (K_{t+1} - K_t)/K_t$, when there is no depreciation of the capital stock or when the adjustment cost function is specified in terms of net investment. Tobin's (marginal) q is defined as $q_t = \mu_t^K / p_t^K$, the ratio of the shadow value of a firm of having an additional unit of installed capital in place to the price of acquiring new capital. Equation (26) can then be rewritten as

$$\frac{I_t}{K_t} = \gamma + \frac{1}{\theta}\left((q_t^K - 1)p_t^K\right) + \omega_t \tag{27}$$

or

$$\frac{I_t}{K_t} = \gamma + \frac{1}{\theta}Q_t + \omega_t \tag{28}$$

where $Q = (q_t - 1)p_t^K$. Since q reflects the expected profitability of a firm's investment relative to the opportunity cost of capital, it can be considered the sufficient statistic for investment. An increase in q signals increased investment; thus investment is an increasing function of q. However, since marginal q is not observable, average q - defined as the ratio of the financial value of a firm to the replacement cost of the existing capital stock - is usually used. Hayashi (1982) shows that if the production function and the adjustment cost function are linear homogenous, then marginal and average q are equal. Notably, our analysis corroborates Fama and French's (1992, 1993) use of the book-to-market ratio as a key determinant of equity returns. Tobin's q determines optimal investment, which in turn determines equity returns. However, since Tobin's (average) q is inversely related to the book-to-market ratio, this implies that the book-to- market ratio determines equity returns.

Now we can investigate the effect of equity returns on physical

investment. This can be done using (20) or (22). However, the nonlinearity of expected returns makes it hard to handle analytically. We take a loglinear approximation method to obtain a solution amenable to easy interpretation.[7] If markets are complete, idiosyncratic risk is uncorrelated with equity returns and hence has no impact on the shadow price of capital. In this case, we can rewrite (20) as

$$
E_t[1+\rho_{t+1}] = \tag{29}
$$
$$
\frac{E_t\left[-\partial G(y_{t+1}, \boldsymbol{w_{t+1}}, K_{t+1}, \tau_{t+1})/\partial K_{t+1} - \partial H(I_{t+1}, K_{t+1}, \omega_{t+1})/\partial K_{t+1} + (1-\delta)\mu_{t+1}^K\right]}{\mu_t^K}
$$

To evaluate this expression, we begin by defining the following relation:

$$
S_{s+1} \equiv \frac{Z_{t+1} + (1-\delta)\mu_{t+1}^K}{\mu_t^K} = \frac{\mu_{t+1}^K}{\mu_t^K}\left(\frac{Z_{t+1}}{\mu_{t+1}^K} + (1-\delta)\right) \tag{30}
$$

where $Z_{t+1} \equiv -\partial G(y_{t+1}, \boldsymbol{w_{t+1}}, K_{t+1}, \tau_{t+1})/\partial K_{t+1} - \partial H(I_{t+1}, K_{t+1}, \omega_{t+1})/\partial K_{t+1}$. Taking logs on both sides of (30), we obtain

$$
\ln S_{t+1} = \ln \mu_{t+1}^K - \ln \mu_t^K + \ln\left(\exp(\ln Z_{t+1} - \ln \mu_{t+1}^K) + (1-\delta)\right) \tag{31}
$$

The last term on the right-hand side of this equation is a nonlinear function of $(\ln Z_{t+1} - \ln \mu_{t+1}^K)$. Taking a first-order Taylor expansion of it about a point $Z/\mu^K = \exp(\ln Z - \ln \mu^K)$, we have

$$
\ln S_{t+1} \approx \chi + \psi \ln \mu_{t+1}^K + (1-\psi)\ln Z_{t+1} - \ln \mu_t^K \tag{32}
$$

---

7) We adopt Campbell's (see Cambell, Lo, and MacKinley, 1997) loglinear approximation procedure of the present value model of the stock price with time-varying expected returns (see also Lettau and Ludvigson, 2002).

where χ and ψ are parameters of linearization defined by $\psi \equiv 1 / \left(Z / \mu^K + (1-\delta) \right)$ and $\chi \equiv - \ln \psi + \psi \ln (1-\delta) + (1-\psi) \ln (1/\psi - 1)$.

Next we take logs on both sides of (29) using (32) and assuming that either variables are jointly lognormally distributed, or applying a second-order Taylor expansion. Then (29) can be written in loglinear form as

$$E_t [\rho_{t+1}] \equiv \psi E_t [\ln \mu^K_{t+1}] + (1-\psi) E_t [\ln Z_{t+1}] - \ln \mu^K_t + \phi_t \quad (33)$$

where ϕ_t consists of linearization constants and variance terms. Equation (33) is a first-order linear difference equation for the log shadow price of capital $(\ln \mu^K)$. Solving this equation forward, applying the law of iterated expectations, and imposing the condition that $\lim_{T \to \infty} E_t \left[\psi^T \ln \mu^T_{t+T} \right] = 0$, we obtain the following expression:

$$\ln \mu^K_t \approx \textstyle\sum_{s=t}^{\infty} \psi^{s-t} \left\{ (1-\psi) E_t [\ln Z_{s+1}] - E_t [\rho_{s+1}] + E_t [\phi_s] \right\} \quad (34)$$

Equation (34) shows that the (log) shadow price of capital is a function of two main discounted components: expected future marginal benefits of capital, $E_t [\ln Z_{s+1}] \ (s = t, t+1, \dots)$ and expected future equity returns, $E_t [\rho_{s+1}] \ (s = t, t+1, \dots)$. An increase in expected future marginal benefits or a decrease in expected future returns raises the shadow value of capital and therefore the optimal rate of investment. We can evaluate the effect of future marginal benefits of capital on the shadow value of capital and hence investment by evaluating $\ln Z_{s+1}$, where $\ln Z_{s+1} = \ln [- \partial G(y_{s+1}, \boldsymbol{w_{s+1}}, K_{s+1}, \tau_{s+1}) / \partial K_{s+1} - \partial H(I_{s+1}, K_{s+1}, \omega_{s+1}) / \partial K_{s+1}] \ (s = t, t+1, \dots)$. We obtain the expression for $- \partial G(y_{s+1}, \boldsymbol{w_{s+1}}, K_{s+1}, \tau_{s+1}) / \partial K_{s+1}$ from (5), and from (25) we derive

$$\frac{\partial (I_{s+1}, K_{s+1}, \omega_{s+1})}{\partial K_{s+1}} = \frac{\theta}{2}\left((\gamma+\omega)-\frac{I_{s+1}}{K_{s+1}}\right)\left((\gamma+\omega)+\frac{I_{s+1}}{K_{s+1}}\right)$$
$$(s = t, t+1, ...) \tag{35}$$

Evaluating $\ln Z_{s+1}$ from these two expressions is complicated because of nonlinearity, but it suggests that changes in future output levels, future variable input prices, future capital stock, and technology as well as adjustment shocks affect the shadow price of capital and therefore optimal investment.

We can evaluate expected future real returns $E_t[\rho_{s+1}]$ $(s = t, t+1, ...)$ in (34) from the asset pricing equation (14). We first note that the Fisher equation yields a conditional relation of the form:

$$E_t[\rho_{s+1}] = E_t[r_{s+1}] + E_t[\pi_{s+1}] \ (s = t, t+1, ...) \tag{36}$$

which says that expected future nominal equity returns are the sum of expected future real returns and the expected future inflation rate. We can evaluate expected future real returns, $E_t[r_{s+1}]$ $(s = t, t+1, ...)$ from (14) by applying the law of iterated expectations. The result gives us

$$
\begin{aligned}
E_t[r_{s+1}] &= E_t[E_s[r_{s+1}]] \\
&= \lambda_{0s+1} + E_t[\beta_{ys+1}\lambda_{ys+1}] + \sum_{j=1}^{J} E_t[\beta_{js+1}\lambda_{js+1}] \\
&\quad + E_t[\beta_{Ks+1}] \ (s = t, t+1, ...)
\end{aligned} \tag{37}
$$

This equation relates expected future real returns to expected future risk premium and therefore to expected future risk factors such as output growth, input price growth, the investment-capital ratio, and technology shocks. Optimal investment responds to changes in expected future risk premium or equity returns. Hence we have

PROPOSITION 5. *Capital investment depends on the expected profitability of capital and expected equity risk premium. Capital investment is positively associated with the expected profitability of capital. An increase in the expected profitability raises optimal investment, while a decrease in the expected profitability lowers optimal investment. However, capital investment is negatively associated with the risk premium or expected future equity returns. An increase in the risk premium or expected future equity returns lowers the shadow price of capital and hence Tobin's Q through a higher rate for discounting future profitability of a firm. As a result, optimal investment falls. Conversely, a decrease in the risk premium or expected future equity returns raises Tobin's Q through a lower discount rate, leading to a rise in optimal investment.*

Most studies on investment typically impose a constant risk premium and have unsuccessfully attempted to explain changes in investment from changes in risk-free rates of interest [Chirinko (1993), Hubbard (1998) for a survey]. However, there is much evidence that expected excess equity returns over a short-term interest rate vary over time and that this variation is much large relative to variation in the risk-free interest rates [Campbell et al. (1997), Cochrane (2001)]. This suggests that most variation in the cost of capital comes from time-varying expected stock returns with relatively constant interest rates, which clearly points to the empirical failure of standard or neoclassical investment studies.

A rise in overall uncertainty or a macroeconomic risk shock such as a war or energy shock increases the equity risk premium, which in turn depresses real economic activity through consumption or investment. A rise in the equity risk premium reduces the stock market value of wealth of consumers, which lowers consumption. Lettau and Ludvigson (2002) show that the consumption channel is not important in transmitting the effects of the risk premium to the real economy because consumers want to maintain relatively flat consumption paths over time. Instead, they present strong evidence that investment growth responds negatively to changes in future equity returns; hence investment is the channel through which a macroeconomic risk shock affects the real economy. From (34), we can see

that a macroeconomic shock reduces the shadow price of capital and hence Tobin's Q, which in turn depresses investment and hence output and the economy.

Because the investment-capital ratio determines equity returns as demonstrated in Section 2, these results clearly suggest a joint or simultaneous link that lies between physical investment and equity returns. The following proposition captures an important finding of this study.

PROPOSITION 6. *The intertemporal equilibrium condition determines equity returns, while a firm's optimal investment decision is derived by adjustment costs. However, investment and equity returns are jointly related to each other. Capital stock growth or the investment-capital ratio determines equity returns. On the other hand, the investment-capital ratio in asset pricing is an endogenous choice variable in a firm's investment decision and is determined by expected future equity returns.*

This result reflects the fundamental difference in methodology between our model and earlier studies [Cochrane (1991, 1996)]. In these studies, adjustment costs drive both optimal investment and equity returns. In fact, in these models, equity returns are determined from the investment equation derived from an adjustment cost technology with a specification of the production function. Thus the model of investment can be equivalently expressed as producing an equity return equation, or the equity return equation yields an investment equation because the two specifications are the same [Cochrane (1991)]. In the present model, in contrast, equity returns are derived from a specification of the short-run variable cost function with capital taken as a quasi- fixed input, as demonstrated in Section 2; we do not need to specify an adjustment cost technology to account for equity returns. An investment equation, on the other hand, requires the specification of an adjustment cost technology. Thus, equity returns and investment decisions are derived from different equations, but they are interrelated. Equity returns are derived from the intertemporal equilibrium condition (23) or (24), but optimal investment is derived from Tobin's Q as described in (28). Nonetheless, while investment determines equity

returns, it responds negatively to changes in the risk premium or future equity returns.

The simultaneity or jointness between equity returns and physical investment may provide some insight into studies that investigate the link between stock returns and real economic activity. Morck, Shleifer, and Vishny (1990) examine how the stock market affects investment and provide evidence that stock returns predict investment growth. Fama (1990), Chen (1991), and Cochrane (1991) find a strong positive relation between stock returns and future production growth rates. However, these studies fail to recognize the simultaneity that exists between stock returns and investment or output growth. In fact, it is argued that "Disentangling cause and effect in the relations between stock returns and real activity is an interesting and formidable challenge ⋯." [Fama (1990)]. As a result, earlier studies often resort to the causality test to determine the direction of a causal link between stock returns and real activity [Peiro (1996)]. Our analysis shows that this practice is not valid because the appropriate procedure is to recognize the simultaneity and to identify relevant variables influencing both stock returns and investment.

Ⅴ. Conclusion

Production variables such as industrial production or GDP and investment growth have often been used to explain equity returns of firms in many studies. However, these studies lack theoretical underpinnings of firm behavior. This paper has provided a microfoundational framework for analyzing the risk-return tradeoff of entrepreneurship and firms. A key risk factor identified in this study to explain equity returns of firms is the investment-capital ratio. This variable has been used in recent studies to account for the cross-sectional variation in equity returns [Cochrane (1996), Li, Vassalou and Xing (2006)]. This study attests to its relevance in asset pricing and showed that it captures the role of firm size and the book-to-market ratio.

Changes in the risk premium or equity returns impinge on a firm's

optimal investment behavior. An important finding of this study is that a joint link exists between equity returns and physical investment: while investment determines equity returns, it responds negatively to changes in the risk premium. Although recent studies recognize the effect of time-varying equity returns on physical investment (Lettau and Ludvigson, 2002), the simultaneous link between the two variables is not well understood. This paper's results should provide useful information about risk analysis, the cost of capital, and investment analysis. Moreover, given the empirical success of the investment-based model, the proposed production-based model may provide a new insight into addressing many issues in consumption-based models such as the equity premium puzzle and the risk-free rate puzzle [Kocherlakota (1996), Cochrane (2001) for a survey]. Finally, it may be noted that our analysis is a partial equilibrium analysis looking at a firm's intertemporal behavior. In a general equilibrium economy, equity returns are determined by consumption as well as production. Also, in general equilibrium, input prices are endogenous, which implies that changes in the risk premium can affect input prices such as wages. Addressing these issues is beyond the scope of the present paper but needs to be explored in future research.

Appendix: The Stochastic Discount Factor Model

The stochastic discount factor model plays a pivotal role in modern analysis of asset pricing [Campbell et al. (1997), Cochrane (2001) for a full discussion]. In fact, most asset pricing models are special cases of the stochastic discount factor model. Our discussion is predicated on this model, so we give a brief summary of it.

We begin with the well-known result:

LEMMA [Cochrane (2001)]. *In the absence of arbitrage opportunities, there exists a positive stochastic discount factor or pricing kernel, $M_{t+1} > 0$, such that, for a traded asset with the price V_t at time t (t = 1, 2, $\cdots$) and a payoff D_{t+1} at time $t+1$, the following condition holds:*

$$V_t = E_t[D_{t+1} M_{t+1}] \tag{A1}$$

or

$$E_t[(1+r_{t+1}) M_{t+1}] = 1 \tag{A2}$$

where $E_t[\,\cdot\,]$ is an expectation conditional on information available to investors at time t, and $r_{t+1} \equiv D_{t+1}/V_t - 1$ is the real rate of return on an asset from time t to time $t+1$.

The stochastic discount factor or pricing kernel is a random variable that assigns prices to cash flows or payoffs to be received in different states of the world. The above relation holds very generally in models that rule out arbitrage opportunities in financial markets. When markets are complete, there exists a unique stochastic discount factor that is consistent with observed asset prices. When markets are incomplete, however, there is more than one stochastic discount factor that satisfies condition (A1) or (A2).

The asset pricing equation (A2) implies bounds on the first and second moments of asset returns and the stochastic discount factor. Expanding the

expectation in (A2), we obtain

$$E_t[1 + r_{t+1}] E_t[M_{t+1}] + Cov_t(r_{t+1}, M_{t+1}) = 1 \qquad \text{(A3)}$$

which yields

$$E_t[1 + r_{t+1}] = \frac{1}{E_t[M_{t+1}]} - \frac{Cov_t(r_{t+1}, M_{t+1})}{E_t[M_{t+1}]} \qquad \text{(A4)}$$

where $Cov_t(\cdot)$ denotes covariance conditional on information at time t. For a risk-free or zero-beta asset with a payoff of one at time $t+1$ that is uncorrelated with the stochastic discount factor, $E_t[M_{t+1}] = 1/(1 + r_{t+1}^f)$, where r_{t+1}^f is the risk-free rate of return from time t to time $t+1$. Assuming the existence of a risk-free asset and expanding the covariance in terms of correlation, we get

$$E_t[r_{t+1}] = r_{t+1}^f - \rho_t(r_{t+1}, M_{t+1}) \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]} \sigma_t(r_{t+1}) \qquad \text{(A5)}$$

where $\rho_t(r_{t+1}, M_{t+1})$ is the correlation coefficient between r_{t+1} and M_{t+1}, and $\sigma_t(M_{t+1})$ and $\sigma_t(r_{t+1})$ are standard deviations of M_{t+1} and r_{t+1}.

Since $|\rho_t(r_{t+1}, M_{t+1})| \leq 1$, equation (A5) implies that the feasible set of means and variances of returns represented by the Sharpe ratio is limited by the volatility of the stochastic discount factor. In particular, we have

PROPOSITION A1 [Hansen and Jagannathan (1991)]. *The Sharpe ratio for any asset places a lower bound on the volatility of the stochastic discount factor given by its standard deviation divided by the conditional mean:*

$$\frac{E_t[r_{t+1}] - r^f_{t+1}}{\sigma_t(r_{t+1})} = -\rho_t(r_{t+1}, M_{t+1}) \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]} \leq \frac{\sigma_t(M_{t+1})}{E_t[M_{t+1}]} \quad \text{(A6)}$$

where $(E_t[r_{t+1}] - r^f_{t+1})/\sigma_t(r_{t+1}))$ *is the Sharpe ratio. The tightest lower bound is achieved by finding the risky asset, or portfolio assets, with the highest Sharp ratio.*

Equation (A6) suggests that the mean-variance frontier of stochastic discount factors that price a given set of assets is related to the mean-variance frontier of asset returns. Hansen and Jagannathan (1991) provide a comprehensive analysis of the volatility bound, allowing for many risky assets and a risk-free asset, and derive implications for the restrictions that the stochastic discount factor must be positive. The stochastic discount factor model allows us to derive multifactor beta pricing models widely used in empirical work in finance [Cochraine (2001)]. Assume that the stochastic discount factor is a linear combination of N common factors f_{nt+1}, $n = 1, ..., N$. For simplicity, assume that the factors have conditional mean zero and are orthogonal to one another. In particular, if

$$M_{t+1} = b_{0t} + \sum_{n=1}^{N} b_{nt} f_{nt+1} \quad \text{(A7)}$$

then

$$E_t[M_{t+1}] = b_{0t} \quad \text{(A8)}$$

$$Cov_t(r_{t+1}, M_{t+1}) = \sum_{n=1}^{N} b_{nt} Cov_t(r_{t+1}, f_{nt+1}) \quad \text{(A9)}$$

It follows:

PROPOSITION A2. *Given the stochastic discount factor* (A7), *one can find a linear multifactor beta asset pricing model:*

$$E_t[r_{t+1}] = \lambda_{0t} + \sum_{n=1}^{N} \beta_{nt+1}\lambda_{nt+1} \qquad (A10)$$

where $\lambda_{0t} \equiv 1/b_{0t}$, $\lambda_{nt+1} \equiv -b_{nt}Var_t(f_{nt+1})/b_{0t}$ $(n=1,...,N)$ is the price of risk of the nth factor and $\beta_{nt+1} \equiv Cov_t(r_{t+1}, f_{nt+1})/Var_t(f_{nt+1})$ $(n=1,...,N)$ is the beta or regression coefficient of an asset return on that factor. Conversely, given the miltifactor beta pricing model (A10), one can find a stochastic discount factor of the form (A7).

Proposition 2 shows the link among the stochastic discount factor, beta pricing, and factor models. It is shown that various asset pricing models amount to alternative ways of applying the stochastic discount factor M_{t+1} to data (see Cochrane, 2001). For the fabled, traditional capital asset pricing model (CAPM), the stochastic discount factor is a linear function of the market portfolio return:

$$M_{t+1} = a_t + b_t r_{t+1}^m \qquad (A11)$$

where r_{t+1}^m is the return on the market portfolio from time t to $t+1$. It follows that

$$Cov_t(r_{t+1}, M_{t+1}) = b_t Cov_t(r_{t+1}, r_{t+1}^m) \qquad (A12)$$

Assuming the existence of a risk-free asset so that $E_t[M_{t+1}] = 1/(1+r_{t+1}^f)$, substitute (A12) into (A4) to obtain

$$E_t[r_{t+1}] = r_{t+1}^f - b_t(1+r_{t+1}^f)Cov_t(r_{t+1}, r_{t+1}^m) \qquad (A13)$$

Since the market portfolio is perfectly correlated with itself, equation (A13) implies that

$$E_t[r^m_{t+1}] = r^f_{t+1} - b_t(1 + r^f_{t+1})\,Var_t(r^m_{t+1}) \tag{A14}$$

Solving this equation for b_t, we have

$$b_t = -\frac{E_t[r^m_{t+1}] - r^f_{t+1}}{(1 + r^f_{t+1})\,Var(r^m_{t+1})} \tag{A15}$$

Substituting this for *bt* allows us to rewrite (A14) as the familiar CAPM:

$$E_t[r_{t+1}] = r^f_{t+1} + \beta_{t+1}(E_t[r^m_{t+1}] - r^f_{t+1}) \tag{A16}$$

where $\beta_{t+1} \equiv Cov_t(r_{t+1}, r^m_{t+1})/Var_t(r^m_{t+1})$. We can see that

$$\lambda_{t+1} \equiv -b_t(1 + r^f_{t+1})\,Var_t(r^m_{t+1}) = E_t[r^m_{t+1}] - r^f_{t+1} \tag{A17}$$

That is, the factor risk price is the market risk premium. Since $\lambda_{t+1} > 0$, equation (A17) implies that $b_t < 0$.

In the consumption-based CAPM, the stochastic discount factor is a consumer's intertemporal marginal rate of substitution in consumption given by the discounted ratio of marginal utilities in two successive periods. For a power or isoelastic specification of the utility function, the ratio of marginal utilities is equal to consumption growth. The risk premium of an asset is determined by covariance between asset returns and consumption growth together with the degree of risk aversion [Campbell et al. (1997), Cochrane (2001)]. The production-based model developed in this paper also belongs to a class of the stochastic discount factor model.

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